UNIT II

1. Sketch qualitatively the influence line for shear at D for the beam

2. Draw the influence line for shear to the left of B for the overhanging beam shown in Fig. Q. No. 4
3. State the position of loading for maximum bending moment at a point in a simply supported beam when it subjected to a series of moving point loads. [M/J-16]

When a series of point loads crosses a simply supported beam, the absolute maximum bending moment will occur near midspan under the load $W_{cr}$, where $W_{cr}$ is the maximum load in a series of point loads.

4. Draw influence line for shearing force at any point in a simply supported beam using Muller Breslau’s principle. [M/J-16]

5. What is the use of influence line diagram (ILD)? [A/M – 12]

Influence lines are very useful in the quick determination of reactions, shear force, bending moment or similar functions at a given section under any given system of moving loads. Influence lines are useful in determining the load position to cause maximum value of a given function in a structure on which load positions can vary.

6. State Muller Breslau’s principle. [N/D – 12,13,14]

Muller-Breslau principle states that, if we want to sketch the influence line for any force quantity (like thrust, shear, reaction, support moment or bending moment) in a structure,

- We remove from the structure the resistant to that force quantity and
- We apply on the remaining structure a unit displacement corresponding to that force quantity.

7. What are influence lines? [N/D - 12, M/J - 14]

An influence line is a graph showing, for any given frame or truss, the variation of any force or displacement quantity (such as shear force, bending moment, tension, deflection) for all positions of a moving unit load as it crosses the structure from one end to the other.

8. Explain the use of Beggs deformeter. [A/M – 11]

It permits extremely accurate work in indirect model analysis.
For best results the deformeter should be used in a room with controlled temperature and humidity so as to avoid disturbance of model deflections due to differential heating. Extended periods of use of this deformeter may cause considerable eye strain.

9. What are the uses of influence lines? [N/D-16]

An influence line is a graph showing, for any given frame or truss, the variation of any force or displacement quantity (such as shear force, bending moment, tension, deflection) for all positions of a moving unit load as it crosses the structure from one end to the other.

10. State: Muller Breslau's principle. [N/D-16][N/D-12 & 13, A/M-12, M/J-14]

Muller-Breslau principle states that, if we want to sketch the influence line for any force quantity (like thrust, shear, reaction, support moment or bending moment) in a structure,

- We remove from the structure the resistant to that force quantity and
- We apply on the remaining structure a unit displacement corresponding to that force quantity

![Diagram of influence line](image)
1. (a) Using Muller Breslau principle, draw the influence line for the bending moment at D, the middle point of span AB of a continuous beam shown Fig. Q. No. 12(a). Compute then ordinates at 1m interval. Determine the maximum hogging bending moment in the beam when two concentrate loads of 8 kN each and separated by a distance 1 m passes though the beam from left to right.

![Diagram](image_url)

**Fig. Q.No. 12 (a)**

**a) Influence lines for *P_A*, *P_B* and *P_C* :**

(i) **IL for *P_A***:

Due to hinge at D, AD will behave as a simple beam. The reaction at D, when it acts upward at D on AD, downward action at D of DBC. Any load on DBC will have no effect on AD.

When a unit load is on AD,

\[ R_A = \frac{4-x}{4} \]

At \( x = 0 \):

\[ R_A = 1 \]

At \( x = 4 \):

\[ R_A = 0 \]

When \( x > 4 \):

\[ R_A = 0 \]

(ii) **IL for *P_B***:

When a unit load is on AD,

\[ R_A = 1 - \frac{x}{4} \]

\[ R_D = 1 - R_A = \frac{x}{4} \]
Taking moment about $C,$
\[ R_B \times 8 - R_D \times 11 = 0 \]
\[ 8 R_B - \frac{7 \times 11}{4} = 0 \]
\[ R_B = \frac{11x}{4 \times 8} = \frac{11x}{32} \]

When $x = 0$ ; $R_B = 0$.
At $x = 4$ ; $R_B = 1.375$

When the load is on $DBC$, the reaction at $D$ is zero.
Taking moment about $C,$
\[ R_B \times 8 - 1 \ (15-x) = 0 \]
\[ R_B = \frac{15-x}{8} \]

When $x = 4$ ; $R_B = 1.375$
$x = 7$ ; $R_B = 1$
$x = 15$ ; $R_B = 0$

(iii) IL for $R_c$ :-
when the unit load is on $AD$, load at $D = \frac{x}{4}$.
taking moments about $B,$
\[ -\frac{x}{4} \times 3 - R_c \times 8 = 0 \]
\[ R_c = -\frac{3x}{32} \]
When $x = 0$ ; $R_B = 0$.
$x = 4$ ; $R_B = -0.375$.

When the load is over $DBC$, $R_D = 0$.
Taking moments about $B,$
\[ -1(17-x) - R_c \times 8 = 0 \]
\[ R_c = \frac{x-7}{8} \]
$x = 4$ ; $R_c = -0.375$
$x = 7$ ; $R_c = 0$
$x = 15$ ; $R_c = 1$
b) I HD for shear to the right of B ($F_B$):

when the load is on $AD$, $F_B = -R_c$

\[ R_c = -\frac{3x}{32}; \quad F_B = \frac{3x}{32} \]

$x = 0 \implies F_B = 0$

$x = 4 \implies F_B = 0.375$

When the load is over $DB$, $F_B = -R_c$

\[ R_c = \frac{x-7}{8}; \quad F_B = \frac{7-x}{8} \]

$x = 4 \implies F_B = 0.375$

$x = 7 \implies F_B = 0$

When the load is over $BC$, $F_B = R_B$

\[ R_B \times 8 = 1(15-x) = 0 \]

\[ F_B = R_B = \frac{15-x}{8} \]

$x = 7 \implies F_B = 1$

$x = 15 \implies F_B = 0$.
(c) B.M. for BM at E (ME):

When unit load is on AD,

\[ ME = P_e \times 6 = -\frac{3x}{32} \times 6 \]

\[ ME = -\frac{18x}{32} \]

\[ x = 0 \implies ME = 0 \]

\[ x = 4 \implies ME = -2.25 \]

When the load is b/n D & E,

\[ ME = P_e \times 6 = \frac{x-7}{8} \times 6 = \frac{3}{8} (x-7) \]

\[ x = 4 \implies ME = -2.25 \]

\[ x = 7 \implies ME = 0 \]

\[ x = 9 \implies ME = 1.5 \]

When the load is b/n E & C:

\[ ME = P_e \times 6 - 1 \times (x-9) \]

\[ P_e = \frac{x-7}{8} \]

\[ ME = (x-7)\frac{6}{8} - (x-9) \]

\[ x = 9 \implies ME = 1.5 \]

\[ x = 15 \implies ME = 0 \]
2. (b) Draw the IL for force in member BC and CI for the truss shown in Figure Q. No. 12(b). The height of the truss is 9 m and each segment is 9 m long.

![Figure Q. No. 12(b)](image)

The nature of shear force in the panel \( L_2L_3 \) changes as the load moves from \( L_2 \) to \( L_3 \).

When unit load is at \( L_2 \), shear force is negative and force in \( u_2L_2 \) is tension. When it is at \( L_3 \), the force in \( u_2L_2 \) is compression.

To find IL0 at \( L_2 \) and at \( L_3 \) and join them to get the IL for shear in the panel \( L_2L_3 \).

Unit load is at \( L_2 \), shear in \( L_2L_3 = -R_B \)

\[ R_A + R_B = 1 \]

Taking moment about B

\[ R_A \times 18 - 1 \times 12 = 0 \]

\[ R_A = \frac{12}{18} = 0.67 \]

\[ R_B = 0.33 \]

Shear in panel, at \( L_2L_3 = -R_B = -0.33 \)

When the load is at \( L_3 \),
\[ RA + RB = 1 \]
\[ RA \times 18 - 1 \times 9 = 0 \]
\[ RA = \frac{9}{18} = 0.5 \]

- Tens at A and B (a) \( L_0 \) and \( L_6 \) are zero.

b) ILD for the member \( U_2, L_3 \):

![Diagram of member \( U_2, L_3 \)]

i) Unit load at \( L_2 \):

Taking moments about \( B \),
\[ RA \times 18 - 1 \times 12 = 0 \]
\[ RA = 0.67 \]

Since \( \Sigma V = 0 \),
\[ RA - 1 - 0.67 \cos 45^\circ = 0 \]
\[ Q = \frac{0.67 - 1}{\cos 45^\circ} = 0.47 \text{ (comp)} \]

ii) Unit load at \( L_3 \):

\[ RA = 0.5 \]

Since \( \Sigma V = 0 \),
\[ RA - 0.5 \cos 45^\circ = 0 \]
\[ Q = 0.707 \text{ (tension)} \]

![Diagram of ILD for member \( U_2, L_2 \)]

![Diagram of ILD for member \( U_2, L_3 \)]
3. (a) A train of loads as shown in Fig. Q. 12 (a) crosses a simply supported beam of 24 m span from left to right. Using influence line determine the maximum bending moment at left one – third span point and also the absolute maximum bending moment in the beam.

The maximum ordinate for negative S.F at \( C = \frac{4}{15} \).

The maximum ordinate for (+ve) S.F at \( C = \frac{11}{15} \).

When 40 kN load is on C,

Negative S.F at \( C = 40 \times \frac{4}{15} + 50 \times \frac{3}{15} + 60 \times \frac{1}{15} \\
= 24.667 \text{ kN.} \)

When 50 kN load is on C,

\((-ve)\) S.F at \( C = \left[ -40 \times \frac{10}{15} + 50 \times \frac{4}{15} + 60 \times \frac{2}{15} + 60 \times \frac{5}{15} \right] \\
= -3.333 \text{ kN.} \)

Maximum \((-ve)\) S.F is when 40 kN lead is on the section and its value is 24.667 kN.
For maximum (+ve) S.F at C, let the 20 kN load to be on C.

\[
\begin{align*}
S.F \text{ at } C &= \left[ 20 \times \frac{11}{15} + 60 \times \frac{9.5}{15} + 60 \times \frac{8}{15} + 50 \times \frac{6}{15} + 40 \times \frac{5}{15} \right] \\
&= 118 \text{ kN.}
\end{align*}
\]

Since, 20 kN is light load, let us try the case, when trailing 60 kN load is on C,

\[
\begin{align*}
S.F \text{ at } C &= \left[ -20 \times \frac{2.5}{15} + 60 \times \frac{11}{15} + 60 \times \frac{9.5}{15} + 50 \times \frac{7.5}{15} + 40 \times \frac{6.5}{15} \right] \\
&= 121 \text{ kN.}
\end{align*}
\]

\[\therefore\] Maximum (+ve) S.F occurs when trailing 60 kN load is on the section = 121 kN.
STEP 3:

\[ y_c = \frac{Z(L - Z)}{L} \]

\[ y_c = 2.933. \]

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Load</th>
<th>Average Load</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( W_{Left} )</td>
<td>( W_{Right} )</td>
</tr>
<tr>
<td>1.</td>
<td>40 kN</td>
<td>190</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{4}{4} )</td>
<td>( \frac{11}{11} )</td>
</tr>
<tr>
<td>2.</td>
<td>50 kN</td>
<td>140</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{4}{4} )</td>
<td>( \frac{11}{11} )</td>
</tr>
<tr>
<td>3.</td>
<td>60 kN</td>
<td>80</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{4}{4} )</td>
<td>( \frac{11}{11} )</td>
</tr>
<tr>
<td>4.</td>
<td>60 kN</td>
<td>20</td>
<td>210</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{4}{4} )</td>
<td>( \frac{11}{11} )</td>
</tr>
</tbody>
</table>

Maximum moment at C,

\[
= 20 \times y_1 + 60 \times y_c + 60 \times y_3 + 50 \times y_3 + 40 \times y_4
\]

\[
= \left[ 20 \times \frac{2.5}{4} + 60 + 60 \times \frac{9.5}{11} + 50 \times \frac{7.5}{15} + 40 \times \frac{6.5}{15} \right] y_c
\]

Maximum B.M = 533.94 kN.m.
4. (b) A Continuous beam ABC is simply resting on supports A and C, and continuous over the support B. The span AB is 6 m and the span BC is 8 m. Draw the influence line diagram for moment at B. Assume Flexural rigidity is constant throughout and calculate the influence line ordinates at 2 m intervals. [M/J-16]
c. Maximum Bending Moment:

\[ \text{Maximum ordinate of ILD (200kN)} = \frac{x(1-x)}{4} \]
\[ = \frac{4x(12-4)}{12} \]
\[ = 2.67 \]

Maximum ordinate at 100kN = \( \frac{2.67 \times 5}{8} = 1.67 \)

Maximum BM = Load \times \text{Ordinate}
\[ = \left[(200 \times 2.67) + (100 \times 1.67)\right] \]
Max. BM = 701 kNm

d. Absolute Maximum Bending Moment:

\[ \text{Max. BM} = 701 \text{ kNm} \]
ordinate under 200kN = \frac{x(1-x)}{1} = \frac{5.5(12-5.5)}{12} = 2.98

ordinate under 100kN = \frac{2.98}{6.5} \times 3.5 = 1.6

Absolute Max. BM = (200 \times 2.98) + (100 \times 1.6) = 756 \text{ kNm}
5. (a) A continuous beam ABC is simply resting on supports A and C. Continuous over the support B and has an internal hinge (D) at 3 m from A. The span AB is 7 m and the span BC is 10 m. Draw influence lines for reactions at A and B.
6. (b) Draw influence line for shearing force at 4m from the propped end of a propped cantilever of span 7m. Calculate the ordinates at every 1m.

**Step 3: Maximum Bending Moment:**

\[ \frac{1}{4} = \frac{16}{4} = 4 \text{ m} \]

Since the udl is acting longer than the span, maximum bending moment occurs right of D.

![Diagram of a propped cantilever with dimensions given: AF 4m, DB 8m, and point D labeled.]

Maximum ordinate at D:

\[ \frac{x(1-x)}{12} = \frac{4(12-4)}{12} \]

\[ = 2.67 \]

Maximum B.M:\[ = \text{load } \times \text{Area} \]

\[ = \frac{1}{2} \times \left[ \frac{1}{2} \times (8 \times 2.67) \right] \]

Max. B.M. = 138.84 kNm.
when $R_B = 1$, then $y_{xB}$ is the displacement at section $x$ due to unit load applied at $B$,

$$N_x = -E I \frac{d^2 y}{dx^2} = R_B \cdot x = 1 \cdot x$$

$$E I \frac{d^2 y}{dx^2} = -x$$

$$E I \frac{dy}{dx} = -\frac{x^2}{2} + c_1 \rightarrow (1)$$

$$E I y = -\frac{x^3}{6} + c_1 x + c_2 \rightarrow (2)$$

At $x = b$,

$$y = 0; \quad \frac{dy}{dx} = 0$$

(1) & (2) we get,

(1)  
$$0 = -\frac{x^2}{2} + c_1$$

$$c_1 = \frac{x^2}{2} = \frac{(b)^2}{2} = 18$$

$$c_1 = 18$$

(2)  
$$0 = -\frac{x^3}{6} + c_1 x + c_2$$

$$c_2 = \frac{x^3}{6} - c_1 x = \frac{(b)^3}{6} - (18 \cdot b)$$

$$c_2 = -72$$

Hence,

$$5 \Rightarrow y_{xB} = \frac{1}{E I} \left[-\frac{x^3}{6} + 18x - 72 \right] \rightarrow (3)$$

$$6 \Rightarrow y_{BB} \ (at \ x = 0) = \left[0 + c_1(0) + c_2\right] \frac{1}{E I}$$

$$y_{BB} = -\frac{72}{E I} \rightarrow (6)$$

IL ordinate for $R_B$ at $x = y_{xB}$

$$\frac{y_{xB}}{y_{BB}} = \frac{1}{E I} \left[-\frac{x^3}{6} + 18x - 72 \right]$$

$$-\frac{72}{E I}$$
7. A live load of 15 kN/m, 5 m long moves on a girder simply supported on a span of 13 m. Find the maximum bending moment that can occur at a section 6 m from the left end.

Step 1: Maximum positive shear force:

ILD for positive ordinate at D = \( \frac{1-x}{1} = \frac{13-6}{13} = 0.54 \)

Ordinate under at C = \( \frac{0.54}{7} \times 2 = 0.15 \)

\[ \text{Maximum positive shear force} = \text{Load} \times \text{Area} = 15 \times \left( \frac{0.54 + 0.15}{2} \times 5 \right) = 25.87 \text{ kN}. \]

Step 2: Maximum negative shear force:

ILD for negative ordinate at C' = \( \frac{x}{1} = \frac{6}{13} = 0.46 \)

Ordinate at C' = \( 0.46 \times \frac{1}{6} = 0.077 \)

Maximum negative SF = \( 15 \times \left( \frac{0.46 + 0.077}{2} \times 5 \right) = 20.14 \text{ kN} \)

Step 3: Maximum bending moment:

Maximum bending moment will occur at D and UDL will be placed at \( \frac{1}{4} \) distance.

\[ l_{\frac{1}{4}} = \frac{5}{4} = 1.25 \text{ m}. \]
8. Explain the procedure and applications of Beggs deformeter. [M/J-14]

Introduced by professor G.E. Beggs of Princeton University in 1922, Beggs' Deformeter addresses all the minute experimental considerations in applying M-B Principle for model analysis.

Fig. 3.25 (b) shows an experimental setup using Beggs' Deformeter. Fig. 3.25 (c) shows a single Beggs' Deformeter gauge.
The gauge is made up of 2 metal bars held together by a pair of spring loaded screws. The bars can be separated by a precise distance with the aid of several pairs of plugs (Fig 3.26). Of the 2 bars, one is called the fixed bar. This has to be fixed to the drawing board with a pair of wood screws. To the other bar, the model, suitably designed and shaped to simulate any given structure, is attached. Three types of connections with the model are possible. (Fig. 3.27).

1. Hinged connection, in which the model is pivoted to the moving bar. A hole in the model engages into a pin on the moving bar.

2. Fixed connection in which the model is clamped to the moving bar using a fixing plate and 4 screws.

3. Floating connection in which neither of the beams is fixed to the drawing board but is kept afloat on a plate of glass, supported on steel balls which rest on another plate of glass. Here the model is attached to both the beams of the gauge using serrated metal strips about 3 mm wide and 25 mm long. To complete the floating gauge connection, the model has to be cut in the region, between the 2 bars of the gauge while normal plugs are in position.

(a) Calibration of plugs

For determining how much displacement is effected by each pair of plugs, normally a cantilever arrangement is employed. The arrangement is shown in Fig. 3.28 (a) to (e). The cantilever can be 100 mm long from the face of fixity to the target point.

The normal position is when both the slots in the gauge are fitted with normal plugs. A micrometer microscope is positioned over the target. This instrument is capable of measuring movements in the $x$ and $y$ co-ordinate directions correct to 0.004 mm. The initial readings are noted with normal plugs in position.

For $x$ displacements we use the 2 pairs of thrust plugs. First remove the normal plugs, and introduce two large thrust plugs. The target points would move in $x$ negative direction. The FM microscope would measure the $x$ movement in the microscope units. (It is not even necessary to convert this into microns or millimeters since the units would cancel off.) Next we introduce two small thrust plugs. This would move the target in $x$-positive direction. The FMM readings would indicate the actual movement due to small thrust plugs. The net movement between the $x$ positive and $x$ negative extremes is the displacement effected by thrust plugs.
To calibrate the moment plugs (o, O),
(i) Observe Y reading of target with normal plugs in place
(ii) Observe Y reading with moment plugs as in Fig 3.28 (e) (small plug above and large plug below). This would cause target to move downwards (Y-negative).
(iii) Observe Y reading with moment plugs inter charged causing upward (Y-positive) movement of target.

The difference between readings in (iii) and (ii) above divided by the length of the cantilever would give the calibration value of moment plugs.

(b) **Filar Micrometer Microscope**

This comes with a heavy metallic stand and can be set up above one target point on the model at a time.

Because of the large magnification what we think is a circular target looks like a figure with jagged edges. So special care must be taken in making targets in the form of black filled in circles.

![Diagram](image)

**Fig. 3.29**

In the field of a f. m. microscope a single diagonal scale serves for both x and y displacements. The diagonal scale is the main scale which is served by an outside drum scale.

The 3 intersecting lines x, y and D can be bodily moved in the field of view of the microscope.

If we want the x movement of the target we first make y line tangential to the target and take the main scale reading on D-line. After the target has shifted, we again bring y line tangential to the new target position shown dotted. This line is marked as y'. Now the D line has moved along the diagonal scale to D'. R' is the new main scale reading. We can use the drum to bring the D' line to the nearest whole main scale reading to get the fraction of the distance from the whole main scale reading.

Recent trends in f.m. microscope is to adopt digital indicators in which the displacement of the target can be read off a monitor attached to the f.m.m.